“The nontrivial zeros of ζ(s) have real part equal to ½.”

Clay Mathematics Institute (Bombieri 2006, 108). Euler’s product encodes the primes. Riemann worked back from the zeta function to their count and linked the zeros to the error in Gauss’s estimate (Bombieri 2006, 109–110).

Where the argument starts

The hypothesis

Mark primes with a one and other integers with a zero. The running total is π(x). Gauss adds the local prime density, roughly 1/ln t, from 2 to x to estimate that total.

The zeta function

Complex weights turn entries through different angles. Taking the logarithm of Euler’s product groups the entries by prime powers.

The search for a proof

Riemann found the analytic connection. Later work excluded regions, placed infinitely many zeros on the line, and checked finite ranges.

Alpöge and Masullo

A zero to the right of ½ would violate the bound Alpöge and Claude prove. In Masullo’s narrower strip, that same zero would fall outside the sum.

What follows

Through 100, count 25 primes. Gauss estimates 29.08. Actual minus estimated is about −4.08 primes. Repeating the subtraction gives the error function $E(x) = \pi(x) - \mathrm{Li}_2(x)$.

Combining the frequency readings keeps the entries through x. Their total is J for log ζ and ψ for $-\zeta'/\zeta$. Pairs of zeros reproduce the steps in ψ.

Those results left room for an off-line zero at an unexamined height. Arithmetic and spectral methods sought a reason that would apply at every height.

Independent checks establish RH and Dirichlet GRH. Cancellation in the proposed general Hecke argument remains an open step.

The November 27 assessment

How large can the error grow?

One fixed multiple of √x ln x bounds |E(x)| at all sufficiently large cutoffs.

At one million

Gauss gives 78,626.50 for 78,498 primes. The signed error is −0.1634 percent of the estimate. Its magnitude divided by √x ln x is 0.00930.

Alpöge and Claude

Verified for the Riemann zeta function and every Dirichlet L-function.

Masullo

Proposed for unitary Hecke L-functions over all number fields. The wider result remains open.

Because Gauss’s estimate grows like x/log x, the absolute deviation as a fraction of that estimate is eventually at most a fixed multiple of $(\log x)^2/\sqrt{x}$. The prime number theorem already gives convergence to zero. RH supplies a rate.

The questions that organize the guide

The snapshot pairs the Clay Mathematics Institute statement with the counting interpretation and the status of each approach. The paper then develops the argument in the order below.

Reader's question	Answer and supporting detail	Paper and microsite
What is the Riemann hypothesis, and why does it matter?	**Riemann linked the zeros of zeta to the gap between the observed prime count and Gauss's estimate.** At 100 we count 25 primes. Gauss estimates 29.08. The ½ in RH gives the square root in the error bound.	p. 2 The hypothesis
What is the Riemann zeta function, and why is it central?	**The zeta function puts prime factorization and counting in the same formula.** Read an input and its output, recover J and ψ at 8½, and watch zero pairs reconstruct the observed steps.	pp. 3–6 The zeta function
What did earlier attempts establish, and what remained open?	**Riemann related counts to zeros. Successive results narrowed where an exception could occur.** Trace the path from Chebyshev and Riemann through zero-free regions, critical-line results, finite checks, and spectral geometry.	pp. 7–8 The search for a proof
How do the Alpöge–Claude and Masullo approaches compare?	**One proof bounds prime-power sums. The other seeks a trace identity over number fields.** Introduce Dirichlet L-functions, identify Claude's contribution, audit Masullo's cancellation step, and define the open number-field question.	pp. 9–13 Alpöge and Masullo

Appendices Worked regression · Transform and proof details · Number fields · Glossary · References

References

Alpöge, Levent. 2026. Note announcing the Riemann hypothesis and its extension to Dirichlet L-functions. November 20.

Bombieri, Enrico. 2006. "The Riemann Hypothesis." In The Millennium Prize Problems, 107–24.

RiemannMasullo Project. 2026. "Lean 4 Formalization of A. M. Masullo, The Music of the Primes." Consulted November 26.

Sutherland, Andrew V. 2026. "Verification Files for the Shorter Lean Proof of RH and of GRH for Dirichlet L-Functions." Updated November 27.